Yr 12 ATAR Physics
Gravitation & Circular Motion Topic Test

Total Marks - 50

NAME: Answers.

1. The latest scientific evidence suggests that the average radius of a hydrogen atom is 5.29×10^{-11} m. Use this current data and information from your data booklet to calculate the velocity with which the electron typically orbits a hydrogen nucleus.

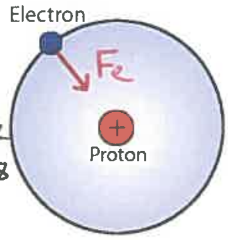
(4 marks)

$$F_e = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$$

$$= \frac{1}{4\pi \times 8.85 \times 10^{-12}} \times \frac{(1.6 \times 10^{-19})^2}{(5.29 \times 10^{-11})^2}$$

$$= 8.99 \times 10^9 \times 9.148 \times 10^{-18}$$

$$= 8.226 \times 10^{-8}\ N$$



since $F_e = F_c = \frac{mv^2}{r}$

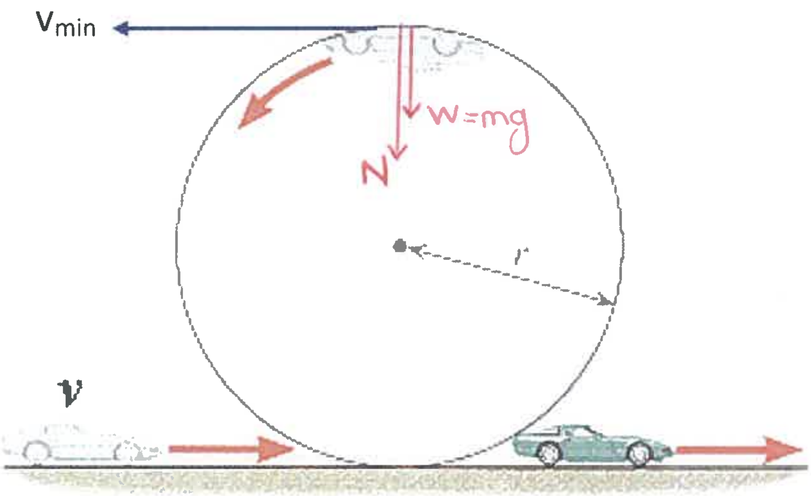
then $8.226 \times 10^{-8} = \frac{9.11 \times 10^{-31} \times v^2}{5.29 \times 10^{-11}}$

$\therefore v^2 = 4.777 \times 10^{12}$

$\therefore v = \sqrt{4.777 \times 10^{12}}$

$= 2.19 \times 10^6\ ms^{-1}$ (tangential to path)

2. A theme park on the Gold Coast has a new attraction whereby a stunt car driver enters a vertical loop of radius, r = 19 m, as shown below.



a). Show all the forces which act on the car at the **top** of the loop.

(2 marks)

b). Calculate the minimum velocity, v_{min} required at the **top** of the loop in order that the driver is able to complete a successful revolution of the track.

(3 marks)

$F_c = N + mg$ $\therefore$ velocity is minimum when $N = 0$!

$\therefore \frac{\cancel{m} v_{min}^2}{r} = \cancel{m} g$ $\therefore V_{min}^2 = rg$

$\therefore V_{min} = \sqrt{(19 \times 9.8)} = 13.6\ ms^{-1}$

c). Hence, calculate the minimum velocity, υ required on **entering** the loop part of the track in order to successfully complete the stunt, assuming that no energy is lost due to friction and the driver does **not** use the car's engine in an attempt to increase her speed through the loop.

Conservation of Energy ⇒ energy at bottom of loop = energy at top of loop **(3 marks)**

$$\therefore \tfrac{1}{2}\cancel{m}\upsilon^2 = \tfrac{1}{2}\cancel{m}V_{min}^2 + \cancel{m}gh$$

$$\therefore \tfrac{1}{2}\upsilon^2 = \tfrac{1}{2} \times 13.6^2 + 9.8 \times 38$$

$$\therefore \tfrac{1}{2}\upsilon^2 = 93.1 + 372.4 = 465.5$$

$$\therefore \upsilon = \sqrt{(2 \times 465.5)} = 30.5\ ms^{-1}.$$

3. A group of students wanted to verify Kepler's laws of planetary motion. They chose to collect data on four of Jupiter's moons: Metis, Adrastea, Amalthea and Thebe.

Kepler's 3rd Law states that: $T^2 = \dfrac{4\Pi^2 r^3}{GM}$ **OR** $r^3 = \dfrac{GMT^2}{4\Pi^2}$

a). **Complete** the following table, using the data provided for the moons.

(2 marks)

Moon	Orbital Radius (r) ($\times 10^6$ m)	Orbital Period (T) ($\times 10^3$ s)	r^3 ($\times 10^{24}$)	T^2 ($\times 10^9$)
Metis	128	25.5	2.10	0.65
Adrastea	129	25.8	2.15	0.67
Amalthea	181	43.0	5.93	1.85
Thebe	222	58.3	10.94	3.40

b). Plot the data from the table above onto the grid provided on the next page in order to demonstrate the relationship described by Kepler's 3rd Law of planetary motion. You must include appropriate units on each axis **and** then draw the line of best fit.

(4 marks)

c). Using your graph, determine Kepler's constant (i.e. the ratio of r^3 to T^2).

(3 marks)

$$\text{gradient} = \frac{r^3}{T^2} = \frac{\text{rise}}{\text{run}} = \frac{(8.5-4) \times 10^{24}}{(2.65-1.25) \times 10^9}$$

$$= \frac{4.5 \times 10^{24}}{1.4 \times 10^9} = 3.21 \times 10^{15}$$

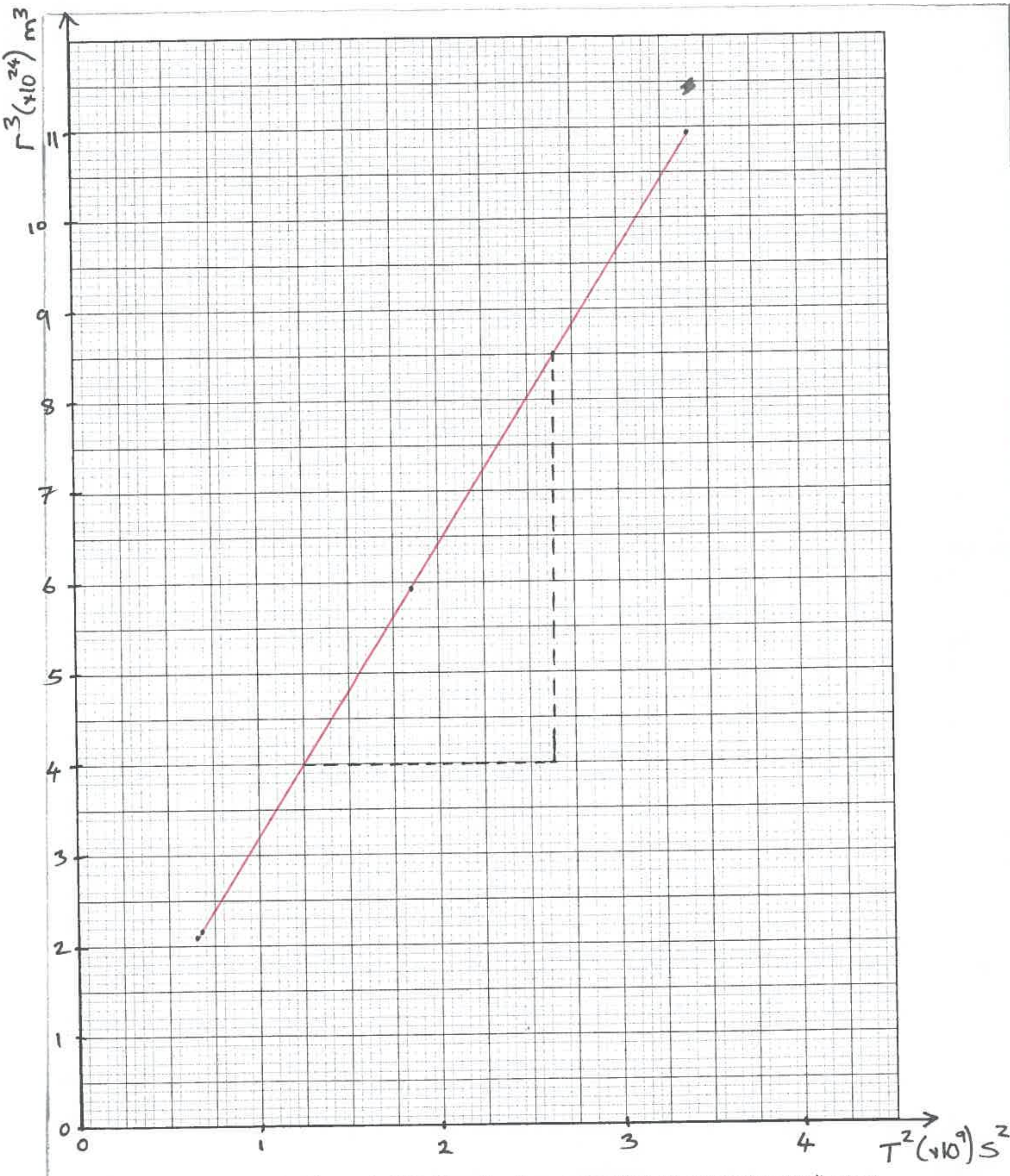
$\dfrac{r^3}{T^2}$ = 3.21×10^{15} $m^3\ s^{-2}$

d). Use your result from part c). to determine the mass of the planet Jupiter.

(3 marks)

$$\frac{r^3}{T^2} = \frac{GM_J}{4\pi^2} \quad \therefore M_J = \frac{\text{gradient} \times 4\pi^2}{G}$$

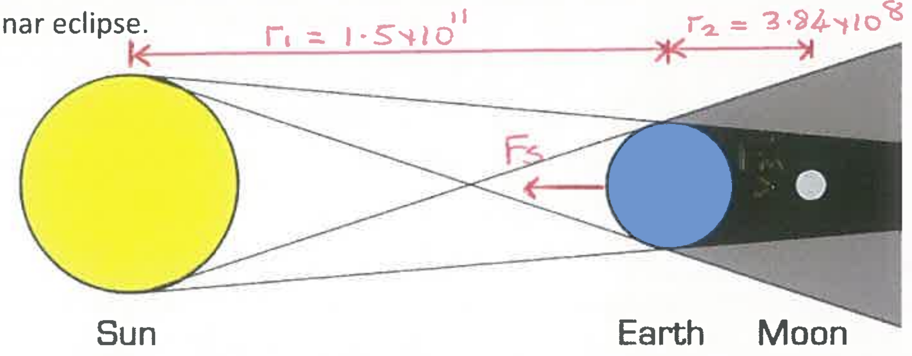
$$\therefore M_J = \frac{3.21 \times 10^{15} \times 4\pi^2}{6.67 \times 10^{-11}} = 1.90 \times 10^{27}\ Kg.$$



A spare grid is provided at the end of this Question/Answer Booklet. If you need to use it, cross out this attempt.

4. Calculate the **resultant** force acting on the Earth due to the moon and the sun during a lunar eclipse.

(4 marks)



for sun on Earth: $F_S = \dfrac{G M_S M_E}{r_1^2} = \dfrac{6.67\times10^{-11}\times 1.99\times10^{30}\times 5.97\times10^{24}}{(1.5\times10^{11})^2}$

$= 3.522\times10^{22}\,N$

for moon on Earth: $F_m = \dfrac{G M_E M_m}{r_2^2} = \dfrac{6.67\times10^{-11}\times 5.97\times10^{24}\times 7.35\times10^{22}}{(3.84\times10^{8})^2}$

$= 1.985\times10^{20}\,N$

$\therefore F_{net} = 3.522\times10^{22} - 1.985\times10^{20} = 3.50\times10^{22}\,N$ towards the Sun.

5. Japan's high-speed bullet train travelling at 288 kmh^{-1} successfully negotiates a turn, which forms part of a horizontal circular path of radius 380 m. The mass of the train, including its passengers is 840 tonnes.

a). Calculate the horizontal stress forces which the track exerts on the train in order to make the turn. $V = 288\,kmh^{-1} = 80\,ms^{-1}$

$m = 840\,t = 840{,}000\,kg$

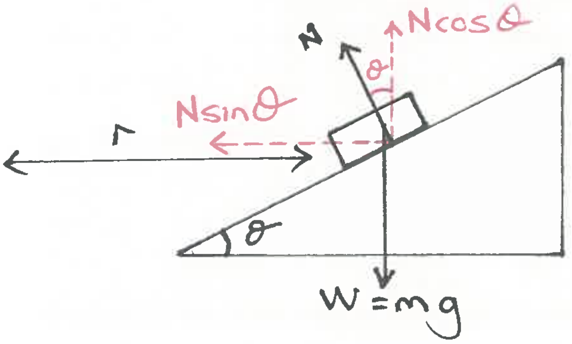
(4 marks)

$F = \dfrac{mv^2}{r} = \dfrac{8.4\times10^5\times 80^2}{380} = 1.41\times10^7\,N$, towards centre of the turn!

b). This section of the track is to be banked, allowing greater control of the train as it makes such turns in the future, without the need to rely on friction and rail stress forces. The equation showing how the angle of banking is related to the speed of the train is:

$$\tan\theta = \frac{v^2}{rg}$$

Use a diagram to explain how this equation can be derived. **(3 marks)**



vertically: $N\cos\theta = mg$ —①

horizontally: $N\sin\theta = F_c = \dfrac{mv^2}{r}$ —②

②/①: $\dfrac{\cancel{N}\sin\theta}{\cancel{N}\cos\theta} = \dfrac{\cancel{m}v^2}{r} \Big/ \cancel{m}g$

$\therefore \tan\theta = \dfrac{v^2}{rg}$

c). Use this equation to calculate the angle at which the track should be banked, given the data provided above. **(2 marks)**

$$\tan\theta = \frac{v^2}{rg} = \frac{80^2}{380 \times 9.8} = 1.72$$

$$\therefore \theta = \tan^{-1}(1.72) = 59.8^\circ$$

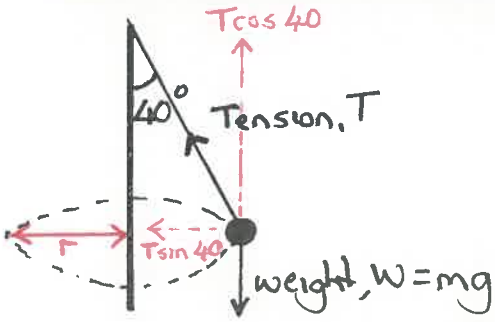
6. A totem tennis game consists of a ball of mass 58.0 g attached to a thick piece of string, length 1.10 m. A child hits the ball which then swings at an angle of 40° to the pole. Calculate:

a). the tension in the string **(2 marks)**

vertically: $T\cos 40 = mg$

$$\therefore T = \frac{0.058 \times 9.8}{\cos 40}$$

$$= 0.742\ \text{N}.$$



b). the speed of the tennis ball after being hit **(3 marks)**

horizontally: $T\sin\theta = \dfrac{mv^2}{r}$;

$$\therefore v^2 = \frac{T\sin\theta\, r}{m} = \frac{0.742 \times \sin 40 \times 0.707}{0.058}$$

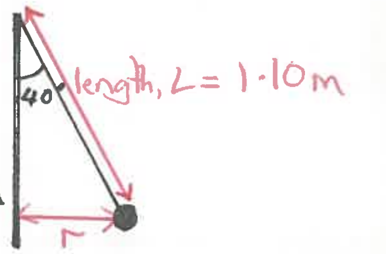
$$\therefore v^2 = 5.814 \qquad \therefore v = \sqrt{(5.814)}$$

$$\therefore v = 2.41\ \text{ms}^{-1}$$

$\sin 40 = \dfrac{r}{1.1}$

$\therefore r = 1.1 \sin 40$

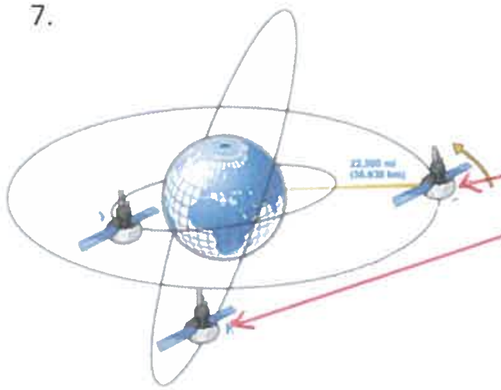
$= 0.707\text{m}$



c). the time it takes the ball to return back to the child **(2 marks)**

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.707}{2.41} = 1.84\ \text{s}.$$

7.



a). On the diagram opposite, label the satellite which is in a polar orbit **and** the satellite which is in a geostationary orbit.

(2 marks)

b). Prove that a geostationary satellite must be placed in an orbit at a height of 3.58 x 10^7 m **above** the Earth's surface.

[Hint: use Kepler's 3rd Law] **(4 marks)**

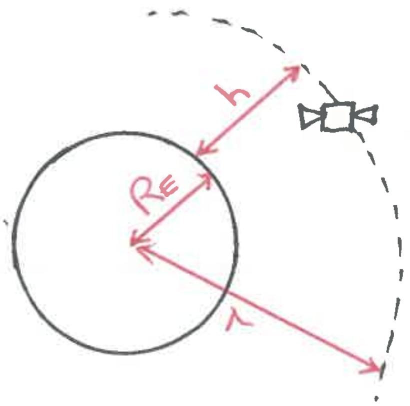
all Geostationary satellites have a Time Period, T= 24hrs.

since $r^3 = \dfrac{GMT^2}{4\pi^2}$,

then $r = \sqrt[3]{\left(\dfrac{6.67\times10^{-11}\times5.97\times10^{24}\times(24\times60\times60)^2}{4\pi^2}\right)}$

$\therefore\ r = \sqrt[3]{\left(\dfrac{2.97\times10^{24}}{4\pi^2}\right)} = \sqrt[3]{(7.53\times10^{22})}$

$\therefore\ r = 4.22\times10^{7}\,m$



height above Earth's surface, $h = r - R_E$

$\therefore\ h = 4.22\times10^{7} - 6.37\times10^{6}$

$= 3.58\times10^{7}\,m.$